ANALYSIS-V (COMPLEX ANALYSIS)

Total marks: 150(Theory: 75, Practical: 50, Internal Assessment: 25)
5 Periods (4 lectures +1 students' presentation),
Practical (4 periods per week per student),

(1st& 2nd Week):
Limits, Limits involving the point at infinity, continuity.
Properties of complex numbers, regions in the complex plane, functions of complex variable, mappings. Derivatives, differentiation formulas, Cauchy-Riemann equations, sufficient conditions for differentiability.
[1]: Chapter 1 (Section 11), Chapter 2 (Section 12, 13) Chapter 2 (Sections 15, 16, 17, 18, 19, 20, 21, 22)

(3rd, 4th & 5th Week):

Analytic functions, examples of analytic functions, exponential function, Logarithmic function, trigonometric function, derivatives of functions, definite integrals of functions.
[1]: Chapter 2 (Sections 24, 25), Chapter 3 (Sections 29, 30, 34),Chapter 4 (Section 37, 38)

(6th Week):
Contours, Contour integrals and its examples, upper bounds for moduli of contour integrals.
[1]: Chapter 4 (Section 39, 40, 41, 43)

(7th Week):
Antiderivatives, proof of antiderivative theorem, Cauchy-Goursat theorem, Cauchy integral formula.
[1]: Chapter 4 (Sections 44, 45, 46, 50)

(8th Week):
An extension of Cauchy integral formula, consequences of Cauchy integral formula, Liouville's theorem and the fundamental theorem of algebra.
[1]: Chapter 4 (Sections 51, 52, 53)

(9th Week):
Convergence of sequences and series, Taylor series and its examples.
[1]: Chapter 5 (Sections 55, 56, 57, 58, 59)

(10th Week):
Laurent series and its examples, absolute and uniform convergence of power series, uniqueness of series representations of power series.
[1]: Chapter 5 (Sections 60, 62, 63, 66)

(11th Week):
Isolated singular points, residues, Cauchy's residue theorem, residue at infinity.
[1]: Chapter 6 (Sections 68, 69, 70, 71)

(12th Week):
Types of isolated singular points, residues at poles and its examples, definite integrals involving sines and cosines.
[1]: Chapter 6 (Sections 72, 73, 74), Chapter 7 (Section 85).

REFERENCES:

1. James Ward Brown and Ruel V. Churchill, Complex Variables and Applications (Eighth Edition), McGraw – Hill International Edition, 2009.

SUGGESTED READING:

1. Joseph Bak and Donald J. Newman, *Complex analysis* (2nd Edition), Undergraduate Texts in Mathematics, Springer-Verlag New York, Inc., New York, 1997.

LAB WORK TO BE PERFORMED ON A COMPUTER (MODELING OF THE FOLLOWING PROBLEMS USING MATLAB/ MATHEMATICA/ MAPLE ETC.)

1. Declaring a complex number and graphical representation.

   e.g. $Z_1 = 3 + 4i$, $Z_2 = 4 – 7i$

2. Program to discuss the algebra of complex numbers.

   e.g., if $Z_1 = 3 + 4i$, $Z_2 = 4 – 7i$, then find $Z_1 + Z_2$, $Z_1 - Z_2$, $Z_1 * Z_2$, and Z_1 / Z_2

3. To find conjugate, modulus and phase angle of an array of complex numbers.

e.g., Z = [2+ 3i   4-2i   6+11i   2-5i]

4. To compute the integral over a straight line path between the two specified end points.

e. g., $\int_C \sin Z\, dz$, where C is the straight line path from -1+ i to 2 - i.

5. To perform contour integration.

e.g., (i) $\int_C (Z^2 - 2Z + 1)dz$, where C is the Contour given by x = y^2 +1; $-2 \le y \le 2$.

(ii) $\int_C (Z^3 + 2Z^2 + 1)dz$, where C is the contour given by $x^2 + y^2 = 1$, which can be

parameterized by x = cos (t), y = sin (t) for $0 \le y \le 2\pi$.

6. To plot the complex functions and analyze the graph .

e.g., (i) f(z) = Z

(ii) f(z)=Z^3

(iii) f(z) = $(Z^4\text{-}1)^{1/4}$

(iv) $f(z) = \overline{z}$, $f(z) = iz$, $f(z) = z^2$, $f(z) = e^z$ etc.

7. To perform the Taylor series expansion of a given function f(z) around a given point z.

The number of terms that should be used in the Taylor series expansion is given for each function. Hence plot the magnitude of the function and magnitude of its Taylors series expansion.

e.g., (i) f(z) = exp(z) around z = 0, n =40.

(ii) f(z)=exp(z^2) around z = 0, n = 160.

8. To determines how many terms should be used in the Taylor series expansion of a given function f(z) around z = 0 for a specific value of z to get a percentage error of less than 5 %.

e.g., For f(z) = exp(z) around z =0, execute and determine the number of necessary terms to get a percentage error of less than 5 % for the following values of z:

(i) z = 30 + 30 i

(ii) $z = 10 + 10\sqrt{3}\ i$

9. To perform Laurents series expansion of a given function f(z) around a given point z.

e.g., (i) f(z)= (sin z -1)/z^4 around z = 0

(ii) f(z) = cot (z)/z^4 around z = 0.

10. To compute the poles and corresponding residues of complex functions.

e.g., $f(z) = \dfrac{z+1}{z^3 - 2z + 2}$

12. To perform Conformal Mapping and Bilinear Transformations.