CALCULUS OF VARIATIONS AND LINEAR PROGRAMMING

Total marks:150 (Theory: 75, Internal Assessment: 25+ Practical: 50)
5 Periods (4 lectures +1 students' presentation),
Practicals(4 periods per week per student)

(1st&2nd Week)
Functionals, Some simple variational problems, The variation of a functional, A necessary condition for an extremum, The simplest variational problem, Euler's equation, A simple variable end point problem.
[1]: Chapter 1 (Sections 1, 3, 4 and 6).

(3rd&4th Week)
Introduction to linear programming problem, Graphical method of solution, Basic feasible solutions, Linear programming and Convexity.
[2]: Chapter 2 (Section 2.2), Chapter 3 (Sections 3.1, 3.2 and 3.9).

(5th& 6th Week)
Introduction to the simplex method, Theory of the simplex method, Optimality and Unboundedness.
[2]: Chapter 3 (Sections 3.3 and 3.4).

(7th& 8th Week)
The simplex tableau and examples, Artificial variables.
[2]: Chapter 3 (Sections 3.5 and 3.6).

(9th&10th Week)
Introduction to duality, Formulation of the dual problem, Primal-dual relationship, The duality theorem, The complementary slackness theorem.
[2]: Chapter 4 (Sections 4.1, 4.2, 4.4 and 4.5).

(11th&12th Week)
Transportation problem and its mathematical formulation, Northwest-corner method, Least-cost method and Vogel approximation method for determination of starting basic solution, Algorithm for solving transportation problem, Assignment problem and its mathematical formulation, Hungarian method for solving assignment problem.
[3]: Chapter 5 (Sections 5.1, 5.3 and 5.4)

PRACTICAL/LAB WORK TO BE PERFORMED ON A COMPUTER: (MODELLING OF THE FOLLOWING PROBLEMS USING EXCEL SOLVER/LINGO/MATHEMATICA, ETC.)

(i) Formulating and solving linear programming models on a spreadsheet using excel solver.
[2]: Appendix E and Chapter 3 (Examples 3.10.1 and 3.10.2).
[4]: Chapter 3 (Section 3.5 with Exercises 3.5-2 to 3.5-5)

(ii) Finding solution by solving its dual using excel solver and giving an interpretation of the dual.
[2]: Chapter 4 (Examples 4.3.1 and 4.4.2)

(iii) Using the excel solver table to find allowable range for each objective function coefficient, and the allowable range for each right-hand side.
[4]: Chapter 6 (Exercises 6.8-1 to 6.8-5).

(iv) Formulating and solving transportation and assignment models on a spreadsheet using solver.
[4]: Chapter 8 (**CASE 8.1**: Shipping Wood to Market, **CASE 8.3**: Project Pickings).

From the Metric space paper, exercises similar to those given below:

1. Calculate d(x,y) for the following metrics

   (i) $X=\mathbf{R}$, $d(x,y)=|x-y|$,

   x: 0, 1, π, e
   y: 1, 2, ½, √2

   (ii) $X=\mathbf{R}^3$, $d(x,y)= (\sum(x_i-y_i)^2)^{1/2}$

   x: (0,1,-1), (1,2,π), (2,-3,5)
   y: (1, 2, .5), (e,2,4), (-2,-3,5)

   (iii) $X=C[0,1]$, $d(f,g)= \sup |f(x)-g(x)|$

   f(x): x^2 , sin x, tan x
   g(x): x , |x|, cos x

2. Draw open balls of the above metrics with centre and radius of your choice.
3. Find the fixed points for the following functions

   $f(x)=x^2$, $g(x)= \sin x$, $h(x)= \cos x$ in $X=[-1, 1]$,

   $f(x,y)= (\sin x, \cos y)$, $g(x,y) = (x^2 , y^2)$ in $X= \{ (x,y): x^2+y^2\leq 1\}$,

under the Euclidean metrics on **R** and $\mathbf{R}^2$ respectively.

4. Determine the compactness and connectedness by drawing sets in R^2.

REFERENCES:

[1]. I. M. Gelfand and S. V. Fomin, *Calculus of Variations*, Dover Publications, Inc., New York, 2000.

[2]. Paul R. Thie and Gerard E. Keough, *An Introduction to Linear Programming and Game Theory*, Third Edition, John Wiley & Sons, Inc., Hoboken, New Jersey, 2008.

[3]. Hamdy A. Taha, *Operations Research*: *An Introduction*, Ninth Edition, Prentice Hall, 2011.

[4]. Frederick S. Hillier and Gerald J. Lieberman, *Introduction to Operations Research*, Ninth Edition, McGraw-Hill, Inc., New York, 2010.

SUGGESTED READING:

[1].R. Weinstock, *Calculus of Variations*, Dover Publications, Inc. New York, 1974.

[2].M. L. Krasnov, G. I. Makarenko and A. I. Kiselev, *Problems and Exercises in the Calculus of Variations*, Mir Publishers, Moscow, 1975.

[3].Mokhtar S. Bazaraa, John J. Jarvis and Hanif D, Sherali, *Linear Programming and Network Flows*, Fourth Edition, John Wiley & Sons, Inc., Hoboken, New Jersey, 2010.

[4].G. Hadley, *Linear Programming*, Narosa Publishing House, New Delhi, 2002.