DIFFERENTIAL EQUATIONS-II (PDE & SYSTEM OF ODE)

Total marks:150 (Theory: 75, Internal Assessment: 25+ Practical: 50)
5 Periods (4 lectures +1 students' presentation),
Practicals(4 periods per week per student)

Section – 1 (1st , 2nd & 3rd Weeks)

Partial Differential Equations – Basic concepts and Definitions, Mathematical Problems. First-Order Equations: Classification, Construction and Geometrical Interpretation. Method of Characteristics for obtaining General Solution of Quasi Linear Equations. Canonical Forms of First-order Linear Equations. Method of Separation of Variables for solving first – order partial differential equations.

[1]: Chapter 1:1.2, 1.3
[1]: Chapter 2: 2.1-2.7

Section – 2 (3-weeks)

Derivation of heat equation, Wave equation and Laplace equation.Classification of second order linear equations as hyperbolic, parabolic or elliptic. Reduction of second order Linear Equations to canonical forms.

[1]: Chapter 3: 3.1, 3.2, 3.5, 3.6
[1]: Chapter 4: 4.1-4.5

Section – 3 (3-weeks)

The Cauchy problem, the Cauchy-Kowaleewskaya theorem, Cauchy problem of an infinite string. Initial Boundary Value Problems, Semi-Infinite String with a fixed end, Semi-Infinite String with a Free end, Equations with non-homogeneous boundary conditions, Non-Homogeneous Wave Equation. Method of separation of variables – Solving the Vibrating String Problem, Solving the Heat Conduction problem

[1]: Chapter 5: 5.1 – 5.5, 5.7
[1]: Chapter 7: 7.1, 7.2, 7.3, 7.5

Section -4 (3-weeks)

Systems of linear differential equations, types of linear systems, differential operators, an operator method for linear systems with constant coefficients, Basic Theory of linear systems in normal form, homogeneous linear systems with constant coefficients: Two Equations in two unknown functions, The method of successive approximations, the Euler method, the modified Euler method, The Runge- Kutta method.

[2]: Chapter 7: 7.1, 7.3, 7.4,

[2]:Chapter 8: 8.3, 8.4-A,B,C,D

REFERENCES:

[1]: TynMyint-U and LkenathDebnath, *Linear Partial Differential Equations for Scientists and Engineers,* 4th edition, Springer, Indian reprint, 2006

[2]: S. L. Ross, *Differential equations,* 3rd Edition, John Wiley and Sons, India,2004

Suggested Reading

[1]: Martha L Abell, James P Braselton, *Differential equations with MATHEMATICA,* 3rd Edition, Elsevier Academic Press, 2004.

LIST OF PRACTICALS
(*MODELLING OF FOLLOWING USING MATLAB/MATHEMATICA/MAPLE*)

1. Solution of Cauchy problem for first order PDE.

2. Plotting the characteristics for the first order PDE.

3. Plot the integral surfaces of a given first order PDE with initial data.

4. Solution of wave equation $\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0$ for any 2 of the following associated conditions:

(a) $u(x,0) = \emptyset(x), u_t(x,0) = \psi(x), \ x \in \mathbb{R}, \ t > 0.$

(b) $u(x,0) = \emptyset(x), u_t(x,0) = \psi(x), \ u(0,t) = 0, \ x \in (0,\infty), \ t > 0.$

(c) $u(x,0) = \emptyset(x), u_t(x,0) = \psi(x), \ u_x(0,t) = 0, \ x \in (0,\infty), \ t > 0.$

(d) $u(x,0) = \emptyset(x)$, $u_t(x,0) = \psi(x)$, $u(0,t) = 0, u(l,t) = 0$, $0 < x < l$, $t > 0$.

5. Solution of one-Dimensional heat equation $u_t = \kappa\, u_{xx}$, for a homogeneous rod of length l.

That is - solve the IBVP:

$$u_t = \kappa u_{xx}, \qquad 0 < x < l, \qquad t > 0,$$

$$u(0,t) = 0, u(l,t) = 0, t \geq 0,$$

$$u(x,0) = f(x), 0 \leq x \leq l$$

6. Solving systems of ordinary differential equations.

7. Approximating solution to Initial Value Problems using any of the following approximate methods:

(a) The Euler Method

(b) The Modified Euler Method.

(c) The Runge-Kutta Method.

Comparison between exact and approximate results for any representative differential equation.

8. Draw the following sequence of functions on given the interval and discuss the pointwise convergence:

(i) $f_n(x) = x^n$ for $x \in \mathrm{R}$, (ii) $f_n(x) = \dfrac{x}{n}$ for $x \in \mathrm{R}$,

(iii) $f_n(x) = \dfrac{x^2 + nx}{n}$ for $x \in \mathrm{R}$, (iv) $f(x) = \dfrac{\sin nx + n}{n}$ for $x \in \mathrm{R}$

(v) $f_n(x) = \dfrac{x}{x+n}$ for $x \in \mathrm{R}, x \geq 0$, (vi) $f_n(x) = \dfrac{nx}{1+n^2x^2}$ for $x \in \mathrm{R}$,

(Vii) $f_n(x) = \dfrac{nx}{1+nx}$ for $x \in \mathrm{R}, x \geq 0$,

(viii) $f_n(x) = \dfrac{x^n}{1+x^n}$ for $x \in \mathrm{R}, x \geq 0$

9. Discuss the uniform convergence of sequence of functions above.