PROBABILITY AND STATISTICS

Total marks:150 (Theory: 75, Internal Assessment: 25+ Practical: 50)
5 Periods (4 lectures +1 students' presentation),
Practicals(4 periods per week per student)
Use of Calculators is allowed

1st Week

Sample space, Probability axioms, Real random variables (discrete and continuous).

2nd Week

Cumulative distribution function, Probability mass/density functions, Mathematical expectation.

3rd Week

Moments, Moment generating function, Characteristic function.

4th Week

Discrete distributions: uniform, binomial, Poisson, Geometric, Negative Binomial distributions.

5th Week

Continuous distributions: Uniform, Normal, Exponential, Gamma distributions
[1]Chapter 1 (Section 1.1, .3, 1.5-1.9)
[2]Chapter 5 (Section 5.1-5.5,5.7), Chapter 6 (Sections 6.2-6.3,6.5-6.6)

6th Week

Joint cumulative distribution Function and its properties, Joint probability density functions – marginal and conditional distributions

7th Week

Expectation of a function of two random variables, Conditional expectations, Independent random variables, Covariance and correlation coefficient.

8th Week

Bivariate normal distribution, Joint moment generating function.

9th Week

Linear regression for two variables, The rank correlation coefficient.

[1]Chapter 2 (Section 2.1, 2.3-2.5)
[2]Chapter 4 (Exercise 4.47), Chapter 6 (Sections 6.7), Chapter 14 (Section 14.1, 14.2), Chapter 16 (Section 16.7)

10thWeek

Chebyshev's inequality, statement and interpretation of (weak) law of large numbers and strong law of large numbers.

11thWeek

Central Limit Theorem for independent and identically distributed random variables with finite variance.

12thWeek

Markov Chains, Chapman – Kolmogorov Equations, Classification of states.
[2] Chapter 4 (Section 4.4)
[3] Chapter 2 (Sections 2.7), Chapter 4 (Section 4.1-4.3)

REFERENCES:

1. Robert V. Hogg, Joseph W. Mc Kean and Allen T. Craig. Introduction of Mathematical Statistics, Pearson Education, Asia, 2007
2. Irvin Miller and Marylees Miller, John E. Freund's Mathematical Statistics with Applications (7thEdn), Pearson Education, Asia, 2006.
3. Sheldon Ross, Introduction to Probability Models (9th Edition), Academic Press, Indian Reprint, 2007

PRACTICALS /LAB WORK TO BE PERFORMED ON A COMPUTER USING SPSS/EXCEL/Mathematica etc.

1. Calculation of
   (i) Arithmetic Mean, geometric Mean, harmonic Mean
   (ii) Variance

2. Fitting of
   (i) Binomial Distribution
   (ii) Poisson Distribution
   (iii) Negative Binomial Distribution
   (iv) Normal Distribution

3. Calculation of
   (i) Correlation coefficients
   (ii) Rank correlation

4. Fitting of polynomials
5. Regression curves
6. Illustrations of Central limit theorem.

7. Draw the following surfaces and find level curves at the given heights:

   (i) $f(x,y)=10-x^2-y^2;\quad z=1,z=6,z=9$,

   (ii) $f(x,y)=x^2+y^2;\quad z=1,z=6,z=9$, (iii) $f(x,y)=x^3-y;\quad z=1,z=6$,

   (iv) $f(x,y)=x^2+\frac{y^2}{4};\quad z=1,z=5,z=8$

   (v) $f(x,y)=4x^2+y^2;\quad z=0,z=1,z=3,z=5$,

   (vi) $f(x,y)=2-x-y;\quad z=-6,z=-4,z=-2,z=0,z=2,z=4,z=6$.

8. Draw the following surfaces and discuss whether limit exits or not as (x,y) approaches to the given points. Find the limit, if it exists:

   (i) $f(x,y)=\frac{x+y}{x-y};\quad (x,y)\to(0,0)$ and $(x,y)\to(1,3)$,

   (ii) $f(x,y)=\frac{x-y}{\sqrt{x^2+y^2}};\quad (x,y)\to(0,0)$ and $(x,y)\to(2,1)$,

   (iii) $f(x,y)=(x+y)e^{xy};\quad (x,y)\to(1,1)$ and $(x,y)\to(1,0)$,

(iv) $f(x,y)=e^{xy}$; $(x,y)\to(0,0)$ and $(x,y)\to(1,0)$,

(v) $f(x,y)=\dfrac{x-y^2}{x^2+y^2}$; $(x,y)\to(0,0)$,

(vi) $f(x,y)=\dfrac{x^2+y}{x^2+y^2}$; $(x,y)\to(0,0)$,

(vii) $f(x,y)=\dfrac{x^2-y^2}{x^2+y^2}$; $(x,y)\to(0,0)$ and $(x,y)\to(2,1)$,

(viii) $f(x,y)=\dfrac{x^2-y}{x+y}$; $(x,y)\to(0,0)$ and $(x,y)\to(1,-1)$.

9. Draw the tangent plane to the following surfaces at the given point:

(i) $f(x,y)=\sqrt{x^2+y^2}$ at $(3,1,\sqrt{10})$, (ii) $f(x,y)=10-x^2-y^2$ at $(2,2,2)$,

(iii) $x^2+y^2+z^2=9$ at $(3,0,0)$,

(iv) $z=\arctan x$ at $(1,\sqrt{3},\dfrac{\pi}{3})$ and $(2,2,\dfrac{\pi}{4})$,

(v) $z=\log|x+y^2|$ at $(-3,-2,0)$.

10. Use an incremental approximation to estimate the following functions at the given point and compare it with calculated value:

(i) $f(x,y)=3x^4+2y^4$ at $(1.01, 2.03)$, (ii) $f(x,y)=x^5-2y^3$ at $(0.98, 1.03)$,

(iii) $f(x,y)=e^{xy}$ at $(1.01, 0.98)$,

(iv) $f(x,y)=e^{x^2y^2}$ at $(1.01, 0.98)$.

11. Find critical points and identify relative maxima, relative minima or saddle points to the following surfaces, if it exist:

(i) $z=x^2+y^2$, (ii) $z=y^2-x^2$, (iii) $z=1-x^2-y^2$, (iv) $z=x^2y^4$.

12. Draw the following regions **D** and check whether these regions are of **Type I** or **Type II**:

(i) $D=\{(x,y)\,|\,0\le x\le 2, 1\le y\le e^x\}$,

(ii) $D=\{(x,y)\,|\,\log y\le x\le 2, 1\le y\le e^2\}$,

(iii) $D=\{(x,y)\,|\,0\le x\le 1, x\le y\le 1\}$,

(iv) The region D is bounded by $y=x^2-2$ and the line $y=x$,

(v) $D=\{(x,y)\,|\,0\le x\le 1, x^3\le y\le 1\}$,

(vi) $D=\{(x,y)\,|\,0\le x\le y^3, 0\le y\le 1\}$,

(vii) $D=\left\{(x,y)\,|\,0\le x\le \frac{\pi}{4}, \cos x\le y\le \sin x\right\}$.