PHYSICS- 15: MATHEMATICAL PHYSICS-IV

The emphasis of the course is on applications in solving problems of interest to physicists. The students are to be examined entirely on the basis of problems, seen and unseen.

Linear Algebra

Vector Spaces: Vector Spaces over Fields of Real and Complex numbers. Examples. Vector space of functions. Linear independence of vectors. Basis and dimension of a vector space. Change of basis. Subspace. Isomorphisms. Inner product and Norm. Inner product of functions: the weight function. Triangle and Cauchy Schwartz Inequalities. Orthonormal bases. Sine and cosine functions in a Fourier series as an orthonormal basis. Gram Schmidt orthogonalisation. (10 Lectures)

Linear Transformations: Introduction. Identity and inverse. Singular and non-singular transformations. Representation of linear transformations by matrices. Similarity transformation. Linear operators. Differential operators as linear operators on vector space of functions. Commutator of operators. Orthogonal & unitary operators and their matrix representations.Adjoint of a linear operator. Hermitian operators and their matrix representation. Hermitian differential operators and boundary conditions. Examples. Eigenvalues & eigenvectors of linear operators. Properties of eigenvalues and eigenvectors of Hermitian and unitary operators. Functions of Hermitian operators/matrices. (14 Lectures)

Tensors: Tensors as multilinear transformations (functionals) on vectors. Examples: Moment of Inertia, dielectric susceptibility. Components of a tensor in a basis. Symmetric and antisymmetric tensors. The completely antisymmetric tensor. Non-orthonormal and reciprocal bases. Summation convention. Inner product of vectors and the metric tensor. Coordinate systems & coordinate basis vectors. Reciprocal coordinate basis. Components of metric in a coordinate basis and association with infinitesimal distance. Change of basis: relation between coordinate basis vectors. Change of tensor components under a change of coordinate system. Example: Inertial coordinates and bases in Minkowski space, Lorentz transformations as coordinate transformations, the Elelctromagnetic tensor and change in its components under Lorentz transformations. (14 Lectures)

Calculus of Variations

Variational Principle: Euler's Equation. Application to Simple Problems(shape of a soap film, Fermat's Principle,etc.). Several Dependent Variables and Euler's Equations. Example: Hamilton's Principle and the Euler-Lagrange equations of motion. Geodesics: geodesic equation as a set of Euler's equations.

Constrained Variations: Variations with constraints. Applications: motion of a simple pendulum, particle constrained to move on a hoop. (10 Lectures)

Suggested Study: (1) Orthogonal polynomials as eigenfunctions of Hermitian differential operators. (2) Determination of the principal axes of moment of inertia through diagonalization. (3) Vector space of wave functions in Quantum Mechanics: Position and momentum differential operators and their commutator, wave functions for stationary states as eigenfunctions of Hermitian differential operator. (4) Lagrangian formulation in Classical Mechanics with constraints. (5) Study of geodesics in Euclidean and other spaces (surface of a sphere, etc). (6) Estimation of ground state energy and wave function of a quantum system.

Reference Books:

- Mathematical Tools for Physics, James Nearing, 2010, Dover Publications
- Mathematical Methods for Physicists, G.B. Arfken, H.J. Weber, F.E.Harris,1970, Elsevier
- Introduction to Matrices and Linear Transformations, D.T. Finkbeiner, 1978, Dover Publications

- Linear Algebra, W. Cheney, E.W. Cheney and D.R. Kincaid, 2012, Jones and Bartlett Learning
- Mathematics for Physicists, Susan M. Lea, 2004, Thomson Brooks/Cole

Additional Books for Reference

- Mathematical Methods for Physicis and Engineers, K.F.Riley, M.P.Hobson, S.J.Bence, 3rd Ed., 2006, Cambridge University Press
- Mathematics for Physicists: Philippe Denneryand Andre Krzywicki, 1967, Dover Publications
- The Mathematics of Physics and Chemistry, H. Margenau and G. M. Murphy, 1956, Van Nostrand
- Advanced Engineering Mathematics, D.J.Zill& W.S.Wright, 4th Ed., 2012,Jones& Bartlett Learning

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PHYSICS PRACTICAL-VII

(Students have to perform at least 5 simulations and 2 experiments from the section VIIA. Additional experiments may be included with approval of the committee of courses)

PHYSICS LAB.-VIIA

Scilab based simulations experiments based on Mathematical Physics problems like

A. Solve differential equations:

   $dy/dx = e^{-x}$ with $y = 0$ for $x = 0$

   $dy/dx + e^{-x}y = x^2$

   $d^2y/dt^2 + 2\, dy/dt = -y$

   $d^2y/dt^2 + e^{-t}dy/dt = -y$

B. Dirac Delta Function:

   Evaluate $\frac{1}{\sqrt{2\pi\sigma^2}}\int e^{\frac{-(x-2)^2}{2\sigma^2}}(x+3)dx$, for $\boldsymbol{\sigma} = 1, 0.1, 0.01$ and show it tends to 5.

C. Fourier Series:

   Program to sum $\sum_{n=1}^{\infty}(0.2)^n$

   Evaluate the Fourier coefficients of a given periodic function (square wave)

D. Frobenius method and Special functions:

   $\int_{-1}^{+1} P_n(\mu)P_m(\mu)d\mu = \delta_{n,m}$

   Plot $P_n(x)$, $j_v(x)$

   Show recursion relation

E. Complex analysis: Integrate $1/(x^2+2)$ numerically and check with computer integration.

F. Integral transform: FFT of e^{-x^2}

G. Linear algebra:

   Multiplication of two 3 x 3 matrices

   Eigenvalue and eigenvectors of

   $$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ 3 & 1 & 4 \end{pmatrix};\begin{pmatrix} 1 & -i & 3+4i \\ +i & 2 & 4 \\ 3-4i & 4 & 3 \end{pmatrix};\begin{pmatrix} 2 & -i & 2i \\ +i & 4 & 3 \\ -2i & 3 & 5 \end{pmatrix}$$

List of Experiments

1. Study of Electron spin resonance- determine magnetic field as a function of the resonance frequency
2. Zeeman effect: with external magnetic field; Hyperfine splitting
3. Quantum efficiency of CCDs
4. To show the tunneling effect in a tunnel diode using forward biased I-V characteristics.
5. Create vacuum in a small chamber using a mechanical (rotary) pump and measure the chamber pressure using a pressure gauge.