Differential Equations

Total marks:150 (Theory: 75, Internal Assessment: 25+ Practical: 50)
5 Periods (4 lectures +1 students' presentation),
Practicals(4 periods per week per student)

(1st , 2nd& 3rd Weeks)

Differential equations and mathematical models.General, particular, explicit, implicit and singular solutions of a differential equation.Exact differential equations and integrating factors, separable equations and equations reducible to this form, linear equation and Bernoulli equations, special integrating factors and transformations.

[2] Chapter 1 (section 1.1, 1.2, 1.4),
[3] Chapter 2 (section 2.1-2.4)

(4th, 5th& 6th Weeks)

Introduction to compartmental model, exponential decay model, lake pollution model (case study of Lake Burley Griffin), drug Assimilation into the blood (case of a single cold pill, case of a course of cold pills), exponential growth of population, limited growth of population, limited growth with harvesting.

[1]Chapter 2 (section 2.1, 2.2, 2.5-2.7),
Chapter 3 (section 3.1-3.3)

(7th, 8th& 9th Weeks)

General solution of homogeneous equation of second order, principle of super position for homogeneous equation, Wronskian: its properties and applications, Linear homogeneous and non-homogeneous equations of higher order with constant coefficients, Euler's equation, method of undetermined coefficients, method of variation of parameters.

[2] Chapter 3 (Section 3.1-3.3, 3.5)

(10th, 11th& 12th Weeks)

Equilibrium points, Interpretation of the phase plane, predatory-prey model and its analysis, epidemic model of influenza and its analysis, battle model and its analysis.
Ref.:
[1]Chapter 5 (Section 5.1-5.3, 5.7) Chapter 6 (Section 6.1-6.4)

REFERENCES:

[1]Belinda Barnes and Glenn R. Fulford, *Mathematical Modeling with case studies, A differential equation approach using Maple and Matlab,* 2nd edition, Taylor and Francis group, London and New York 2009.

[2] C.H. Edwards and D.E. Penny, *Differential equations and boundary value problems Computing and modelling,* Pearson Education India, 2005.

[3] S.L. Ross, *Differential equations* 3rd edition, John Wiley and Sons, India,2004.

Suggestive Reading

[1] Martha L Abell, James P Braselton, *Differential equations with MATHEMATICA,* 3rd Edition, Elsevier Academic Press, 2004.

LIST OF PRACTICALS
(*MODELLING OF FOLLOWING USING MATLAB/MATHEMATICA/MAPLE*)

1. Plotting of second order solution family of differential equation.
2. Plotting of third order solution family of differential equation.
3. Growth model (exponential case only).
4. Decay model (exponential case only).
5. Any two of the following:
   a) Lake pollution model (with constant/seasonal flow and pollution concentration).
   b) Case of single cold pill and a course of cold pills.
   c) Limited growth of population (with and without harvesting).
6. Any two of the following:
   a) Predatory-prey model (basic volterra model, with density dependence, effect of DDT, two prey one predator).
   b) Epidemic model of influenza (basic epidemic model, contagious for life, disease with carriers).
   c) Battle model (basic battle model, jungle warfare, long range weapons).

7. Plotting of recursive sequences.

8. Find a value of N that will make the following inequality holds for all $n > N$:

   (i) $|\sqrt[n]{0.5}-1| < 10^{-3}$, (ii) $|\sqrt[n]{n}-1| < 10^{-3}$,

(ii) $(0.9)^n < 10^{-3}$, (iv) $2^n / n! < 10^{-7}$ etc.

9. Study the convergence of sequences through plotting.

10. Verify Bolzano Weierstrass theorem through plotting of sequences and hence identify convergent subsequences from the plot.

11. Study the convergence/divergence of infinite series by plotting their sequences of partial sum.

12. Cauchy's root test by plotting nth roots.

13. Ratio test by plotting the ratio of nth and n+1th term.

14. For the following sequences $< a_n >$, given $\in > 0$ and $p \in N$,

Find $m \in N$ such that $(i) | a_{m+p} - a_m | < \in, \quad (ii) | a_{2m+p} - a_{2m} | < \in$

(a) $a_n = \dfrac{n+1}{n}$ for $(\in = \dfrac{1}{2^k},\ p = 10^j\ ,\ j = 1,2,3,4....), k = 0,1,2,5...$

(b) $a_n = \dfrac{1}{n}$ for $(\in = \dfrac{1}{2^k},\ p = 10^j\ ,\ j = 1,2,3,4....), k = 0,1,2,5...$

(c) $a_n = 1 + \dfrac{1}{2!} + ... + \dfrac{1}{n!}$ for $(\in = \dfrac{1}{2^k},\ p = 10^j\ ,\ j = 1,2,3...), k = 0,1,2,..$

(d) $a_n = \dfrac{(-1)^n}{n}$ for $(\in = \dfrac{1}{2^k},\ p = 10^j\ ,\ j = 1,2,3,4....), k = 0,1,2,5...$

(e) $a_n = 1 - \dfrac{1}{2} + \dfrac{1}{3} - .... + \dfrac{(-1)^n}{n}$ for $(\in = \dfrac{1}{2^k},\ p = 10^j\ ,\ j = 1,2,3...), k = 0,1,2,..$

15. For the following series $\sum a_n$, calculate

$(i) \left| \dfrac{a_{n+1}}{a_n} \right|, (ii) |a_n|^{\frac{1}{n}}$, for $n = 10^j,\ j = 1,2,3,.....$, and identify the

convergent series:

(a) $a_n = \left(\frac{1}{n}\right)^{1/n}$ (b) $a_n = \frac{1}{n}$ (c) $a_n = \frac{1}{n^2}$ (d) $a_n = \left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$

(e) $a_n = \frac{n!}{n^n}$ (f) $a_n = \frac{n^3 + 5}{3^n + 2}$ (g) $a_n = \frac{1}{n^2 + n}$ (h) $a_n = \frac{1}{\sqrt{n+1}}$

(j) $a_n = \cos n$ (k) $a_n = \frac{1}{n \log n}$ (l) $a_n = \frac{1}{n(\log n)^2}$