III.2:Numerical Methods

Total marks:150 (Theory: 75, Internal Assessment: 25+ Practical: 50)
5 Periods (4 lectures +1 students' presentation),
Practicals(4 periods per week per student)
Use of Scientific Calculators is allowed.

Algorithms, Convergence, Errors: (I & II weeks)
Relative, Absolute, Round off, Truncation.
[1] 1.1, 1.2
[2] 1.3 (pg 7-8)

Transcendental and Polynomial equations: (III & IV weeks)
Bisection method, Newton's method, Secant method.Rate of convergence of these methods.
[2] 2.2, 2.3, 2.5, 2.10

System of linear algebraic equations (V & VI weeks)
Gaussian Elimination and Gauss Jordan methods.Gauss Jacobi method, Gauss Seidel method and their convergence analysis.
[2]   3.1, 3.2, 3.4

Interpolation: (VII & VIII weeks)
Lagrange and Newton's methods.Error bounds.
Finite difference operators. Gregory forward and backward difference interpolation.
[2] 4.2, 4.3, 4.4

Numerical Integration: (IX & X weeks)
Trapezoidal rule, Simpson's rule, Simpsons $3/8^{th}$ rule, Boole's Rule.
Midpoint rule, Composite Trapezoidal rule, Composite Simpson's rule.
[1] 6.4, 6.5 (pg 467- 482)

Ordinary Differential Equations: XI & XII weeks)
Euler's method.Runge-Kutta methods of orders two and four.
[1] 7.2 (pg 558 - 562), 7.4

REFERENCES:
1. Brian Bradie, A Friendly Introduction to Numerical Analysis, Pearson Education, India, 2007.
2. M. K. Jain, S. R. K. Iyengar and R. K. Jain, Numerical Methods for Scientific and Engineering Computation, New age International Publisher, India, 6th edition, 2007.

SUGGESTED READING:
1. C. F. Gerald and P. O. Wheatley, App;ied Numerical Analysis, Pearson Education, India,7th edition,(2008)
2. Uri M. Ascher and Chen Greif : A first course in Numerical Methods, PHI Learning Private Limited, (2013).
3. John H. Mathews and Kurtis D. Fink: Numerical Methods using Matlab, 4th Edition, PHI Learning Private Limited(2012).

LIST OF PRACTICALS

Practical / Lab work to be performed on a computer: Use of computer aided software (CAS), for example Matlab / Mathematica / Maple / Maxima etc., for developing the following Numerical programs:

(i) Bisection Method
(ii) Secant Method
(iii) Newton Raphson Method
(iv) Gauss-Jacobi Method
(v) Gauss-Seidel Method
(vi) Lagrange Interpolation
(vii) Newton Interpolation
(viii) Composite Simpson's Rule
(ix) Composite Trapezoidal Rule
(x) Euler's Method
(xi) RungeKutta Method of order 2 and 4.

Illustrations of the following :

1. Let $f(x)$ be any function and L be any number. For given a and $\in > 0$, find a $\delta > 0$ such that for all x satisfying $0 < |x-a| < \delta$, the inequality $|f(x)-L| < \in$ holds. For examples:

(i) $f(x) = x+1, L = 5, a = 4, \in = 0.01$

(ii) $f(x) = \sqrt{x+1}, L = 1, a = 0, \in = 0.1$

(iii) $f(x)=x^2, L=4, a=-2, \in=0.5$

(iv) $f(x)=\frac{1}{x}, L=-1, a=-1, \in=0.1$

2. Discuss the limit of the following functions when x tends to 0: $\pm\frac{1}{x}, \sin\frac{1}{x}, \cos\frac{1}{x}, x\sin\frac{1}{x}, x\cos\frac{1}{x}, x^2\sin\frac{1}{x}, \frac{1}{x^n}(n\in N), |x|, [x], \frac{1}{x}\sin x$.

3. Discuss the limit of the following functions when x tends to infinity : $e^{\frac{1}{x}}, e^{-\frac{1}{x}}, \sin\frac{1}{x}, \frac{1}{x}e^x, \frac{1}{x}e^{-x}, \frac{x}{1+x}, x^2\sin\frac{1}{x}, \frac{ax+b}{cx^2+dx+e}(\text{a}\neq 0\neq \text{c})$

4. Discuss the continuity of the functions at $x=0$ in practical 2.

5. Illustrate the geometric meaning of Rolle's theorem of the following functions on the given interval :

(i) x^3-4x on $[-2, 2]$, (ii) $(x-3)^4(x-5)^3$ on $[3, 5]$ etc.

6. Illustrate the geometric meaning of Lagrange's mean value theorem of the following functions on the given interval:

(i) $\log x$ on $\left[1/2, 2\right]$, (ii) $x(x-1)(x-2)$ on $\left[0, 1/2\right]$,

(iii) $2x^2-7x+10$ on $[2, 5]$ etc.

7. For the following functions and given $\in>0$, if exists, find $\delta>0$ such that $|f(x_1)-f(x_2)|<\in$ whenever $|x_1-x_2|<\delta$, and discuss uniform continuity of the functions:

   (i) $f(x)=\frac{1}{x}$ on $[0, 5], \in=\frac{1}{2^j}, j=0,1,2,3,...$

   (ii) $f(x)=\frac{1}{x}$ on $(0, 5], \in=\frac{1}{2^j}, j=0,1,2,3,...$

   (iii) $f(x)=x^2$ on $[-1, 1], \in=\frac{1}{2^j}, j=0,1,2,3,...$

   (iv) $f(x)=\sin x$ on $(0, \infty), \in=\frac{1}{2^j}, j=0,1,2,3,...$

   (v) $f(x)=\sin x^2$ on $(0, \infty), \in=\frac{1}{2^j}, j=0,1,2,3,...$

(vi) $f(x)=\frac{x}{1+x^2}$ on $\mathrm{R}, \in=\frac{1}{2^j}, j=0,1,2,3,\ldots$

(vii) $f(x)=x^3$ on $[0,\ 1], \in=\frac{1}{2^j}, j=0,1,2,3,\ldots$

8. Verification of Maximum –Minimum theorem, boundedness theorem & intermediate value theorem for various functions and the failure of the conclusion in case of any of the hypothesis is weakened.
9. Locating points of relative & absolute extremum for different functions
10. Relation of monotonicity & derivatives along with verification of first derivative test.
11. Taylor's series - visualization by creating graphs:
    a. Verification of simple inequalities
    b. Taylor's Polynomials – approximated up to certain degrees
    c. Convergence of Taylor's series
    d. Non-existence of Taylor series for certain functions
    e. Convexity of the curves

Note: For any of the CAS Matlab / Mathematica / Maple / Maxima etc., the following should be introduced to the students.
Data types-simple data types, floating data types
arithmetic operators and operator precedence,
variables and constant declarations, expressions, input/output,
relational operators, logical operators and logical expressions,
control statements and loop statements, arrays.