Semester-IV

PAPER NO-8: REAL ANALYSIS AND NUMERICAL ANALYSIS

I REAL ANALYSIS

1. Completeness
   - 1.1 The Completeness property of R
2. Neighbourhood and Limit Points
   - 2.1 Neighbourhood
   - 2.2 Open Set
   - 2.3 Closed Set
   - 2.4 Limit Point of a Set
3. Sequences
   - 3.1 Definition of a Sequence
   - 3.2 Convergent Sequence
   - 3.3 Divergent Sequence
   - 3.4 Oscillatory Sequence
   - 3.5 Cauchy Sequence
   - 3.6 Monotone Sequence
4. Series
   - 4.1 Sequence of partial sum of series
   - 4.2 Cauchy's general principal of convergence
   - 4.3 Necessary condition for convergence
   - 4.4 Series of positive terms
   - 4.5 Comparison Tests
   - 4.6 p- test
   - 4.7 Cauchy's n^{th} root test
   - 4.8 D' Alembert's Ratio test
   - 4.9 Raabe's test
   - 4.10 Series of arbitrary terms
   - 4.11 Alternating series
   - 4.12 Absolute convergence
   - 4.13 Conditional convergence
5. Mean Value Theorems
   - 5.1 Rolle's Theorem
   - 5.2 Lagrange's Mean Value theorem
   - 5.3 Power series expansion of e^x, sinx, cosx and log(1+x)

II NUMERICAL ANALYSIS

6. Finite Differences and interpolation with equal intervals
   - 6.1 Definition of operators and derivation of inter-relations among them
   - 6.2 Newton–Gregory formula for forward Interpolation
   - 6.3 Factorial notation for positive and negative exponent. Representation of polynomial in factorial notations
   - 6.4 Properties of Δ and E (without proof)
7. Interpolation with unequal intervals of the argument
   - 7.1 Definition of divided differences of order n.
   - 7.2 Properties of divided differences (only statement).
   - 7.3 Derivation of Newton's divided difference formula
     - 7.3.1 Derivation of Newton's forward difference formula

7.3.2 Newton's backward difference formula
7.4 Lagrange's interpolation formula
7.5 Properties of Lagrangian coefficients (only statement)
8. Central Difference Interpolation formula
8.1 Derivation of Gauss's Forward central difference interpolation formula
8.2 Statement of Gauss's backward central difference interpolation formula
8.3 Derivation of Sterling's central difference interpolation formula
9. Inverse Interpolation
9.1 Lagrange's inverse interpolation formula
9.2 Method of successive approximation or iteration
9.3 Method of reversion of series
10. Numerical Integration
10.1 Derivation of Newton Cotes integration formula
10.2 Properties of Cotes numbers (only statement)
10.3 Derivation of Trapezoidal rule with error term and its geometrical significance
10.4 Derivation of Simpson's one-third rule with error term and its geometrical significance
10.5 Derivation of Simpson's three-eighth rule with error terms and its geometrical significance
10.6 Statement of Weddle's rule
10.7 Derivation of Euler-Maclaurin's summation formula
11. Difference Equation
11.1 Solution of difference equations of first order with variable coefficients
11.2 Solution of linear difference equations with constant coefficients

WEEK-WISE DETAILS

Week 1: Completeness, Neighbourhood and Limit Points

- Ross, Kenneth A. (2007): *Elementary Analysis: The Theory of calculus,* 2nd Edition. pp 19-22
- Bartle, Robert G. and Donald R. Sherbert (2000)*: Introduction to Real Analysis,* 3rd Edition, John Wiley & Sons, Inc. pp. 34-38., 312-314

Week 2-3: Sequences

- Ross, Kenneth A. (2007): *Elementary Analysis: The Theory of calculus*, 2nd Edition. pp 31,54,57,60
- Bartle, Robert G. and Donald R. Sherbert (2000)*: Introduction to Real Analysis,* 3rd Edition, John Wiley & Sons, Inc. pp. 52-54, 73, 80-82

Week 4-5: Series

- Bartle, Robert G. and Donald R. Sherbert (2000)*: Introduction to Real Analysis,* 3rd Edition, John Wiley & Sons, Inc. pp. 89-93, 253-254, 257-260, 263-264.

Week 6: Mean Value Theorems

- Ross, Kenneth A. (2007): *Elementary Analysis: The Theory of calculus,* 2nd Edition. pp 214-215
- Singal, M.K. and Singal, A.R. (2003): *A First Course in Real Analysis,* 24th Edition. pp 328-333.

Week 7: Finite Differences and interpolation with equal intervals

- Kelly, Louis G. (1967): *Handbook of Numerical Methods and Applications*, Addison-Wesley Publishing Company. pp. 23-29.
- Saxena, H.C. (2005): *Finite Differences and Numerical Analysis,* 15th Edition, Sultan Chand and Co. pp. 8-13, 17, 19.
- Scarborough, James B. (1966): *Numerical Mathematical Analysis*, 6th Edition, Oxford IBH Publishing Company Pvt. Ltd. pp. 46-47, 54-61.

Week 8: Interpolation with unequal intervals of the argument

- Kelly, Louis G. (1967): *Handbook of Numerical Methods and Applications*, Addison-Wesley Publishing Company, pp. 30-32, 36-39.

Week 9-10: Central Difference interpolation formulas, Inverse Interpolation

- Hildebrand, F.B. (1987): *Introduction to Numerical Analysis*, 2nd Edition, Tata McGraw-Hill. pp. 136-140
- Scarborough, James B.(1966): *Numerical Mathematical Analysis*, 6th Edition, Oxford IBH Publishing Company Pvt. Ltd. pp. 93-98

Week 11: Numerical Integration

- Kelly, Louis G. (1967): *Handbook of Numerical Methods and Applications*, Addison-Wesley Publishing Company, pp. 52-55
- Brian Bradie (2007): *A Friendly Introduction to Numerical Analysis*, Pearson Education, India. pp 467-471.

Week 12: Difference Equations

- Saxena, H.C.(2005): *Finite Differences and Numerical Analysis,* 15th Edition, Sultan Chand and Co. pp. 173-199

Practical/ Lab work

LIST OF PRACTICALS

1. Formation of difference table, fitting of polynomial and missing terms for equal interval of differencing
2. Based on Newton's Gregory forward difference interpolation formula
3. Based on Newton's backward difference interpolation formula.
4. Based on Newton's divided difference and Lagrange's interpolation formula
5. Based on Gauss forward, Gauss backward central difference interpolation formula
6. Based on Stirling's central difference interpolation formula
7. Based on Lagrange's Inverse interpolation formula
8. Based on method of successive approximation or iteration
9. Based on method of reversion of series
10. Based on Trapezoidal Rule
11. Based on Simpson's one-third rule
12. Based on Simpson's three-eighth rule
13. Based on Weddle's rule
14. To find sum $\sum_a^b f(x)$ by Euler-Maclaurin summation formula
15. To find $\int_a^b f(x)\,dx$ using Euler-Maclaurin summation formula